

Meta-Stable Brane Configurations by Dualizing the Two Gauge Groups

Changhyun Ahn

Department of Physics, Kyungpook National University, Taegu 702-701, Korea

ahn@knu.ac.kr

Abstract

We consider the $\mathcal{N} = 1$ supersymmetric gauge theories with product gauge groups. The two kinds of D6-branes in the electric theory are both displaced and rotated respectively where these deformations are interpreted as the mass terms and quartic terms for the two kinds of flavors. Then we apply the Seiberg dual to the whole gauge group factors by moving the branes and obtain the corresponding dual gauge theories. By analyzing the magnetic superpotentials consisting of an interaction term between a magnetic meson field and dual matters as well as the above deformations for each gauge group, we present the type IIA nonsupersymmetric meta-stable brane configurations.

1 Introduction

The dynamical supersymmetry breaking in meta-stable vacua [1, 2] arises in the $\mathcal{N} = 1$ supersymmetric gauge theory with massive fundamental quarks. The additional quartic term for the quarks in the electric superpotential [3, 4] also leads to the nonsupersymmetric meta-stable ground states in its magnetic theory when the gravitational attraction of NS5-brane [5] is considered. In this construction, taking the Seiberg dual, magnetic theory, from an electric theory is a crucial step to find out new meta-stable supersymmetry breaking vacua.

So far, the Seiberg dual one takes in the context of nonsupersymmetric meta-stable ground states is only for a single gauge group from a single or multiple electric gauge group. Although the electric theory has many gauge group factors, the magnetic dual only for one single gauge group is considered. On the other hand, it is known, in the construction of supersymmetric ground states or its type IIA brane configurations [6], that a number of gauge theory duals(magnetic theory) involving product gauge groups can be interpreted in terms of branes of type IIA string theory. Then it is natural to ask what happens for the dynamical supersymmetry breaking in meta-stable vacua, in the $\mathcal{N} = 1$ supersymmetric product gauge theories which have mass terms [1, 7, 8, 9] and quartic terms [3, 4] for the flavors in the electric superpotential, if one dualizes the whole two gauge groups, not a single gauge group.

One simplest example can be realized by three NS-branes, D4-branes and D6-branes [10]. Now the second example can be obtained by adding orientifold 4-plane to this brane configuration and describes different gauge group and matter contents [11, 12]. Or if one adds orientifold 6-plane to the simplest brane configuration, one possible third example is realized by four NS-branes, D4-branes and D6-branes [13]. All of these examples possess their Seiberg duals either in the gauge theory side [14, 15, 13] or string theory side for the supersymmetric ground states 10 years ago.

In this paper, one reexamines these supersymmetric brane configurations and extracts the possible brane motions, during the dual process, for new meta-stable brane configurations, along the lines of [16, 17, 18, 19, 20]. The geometrical positions of the branes and the creation of D4-branes when the NS5-brane and D6-brane are intersecting each other with an angle, play the important role for removing the unwanted gauge singlets and selecting the wanted gauge singlet which is originated from the quadratic term and mass term of flavors in an electric theory.

In section 2, we review the type IIA brane configuration corresponding to the $\mathcal{N} = 1$ $SU(N_c) \times SU(N'_c)$ gauge theory with fundamentals and bifundamentals and deform this theory by adding both the mass terms and the quartic terms for the fundamentals. Then we describe

the dual $\mathcal{N} = 1$ $SU(\tilde{N}_c) \times SU(\tilde{N}'_c)$ gauge theory with corresponding dual matter as well as a gauge singlet. We discuss the nonsupersymmetric meta-stable minimum and present the corresponding intersecting brane configuration of type IIA string theory.

In section 3, we review the type IIA brane configuration corresponding to the $\mathcal{N} = 1$ $SO(2N_c) \times Sp(N'_c)$ gauge theory with vectors, fundamentals, and bifundamentals and deform this theory by adding both the mass terms and the quartic terms for the vectors and fundamentals. Then we describe the dual $\mathcal{N} = 1$ $SO(2\tilde{N}_c) \times Sp(\tilde{N}'_c)$ gauge theory with corresponding dual matter as well as a gauge singlet. We describe the nonsupersymmetric meta-stable minimum and the corresponding intersecting brane configuration of type IIA string theory.

In section 4, we review the type IIA brane configuration corresponding to the $\mathcal{N} = 1$ $SU(N_c) \times SO(N'_c)$ gauge theory with fundamentals, vectors, and bifundamentals and deform this theory by adding both the mass terms and the quartic terms for the fundamentals and vectors. Then we describe the dual $\mathcal{N} = 1$ $SU(\tilde{N}_c) \times SO(\tilde{N}'_c)$ gauge theory with corresponding dual matter as well as a gauge singlet. We study the nonsupersymmetric meta-stable minimum and the corresponding intersecting brane configuration of type IIA string theory.

In section 5, we comment on the future directions.

2 $SU(N_c) \times SU(N'_c)$ with N_f - and N'_f -fund. and bifund.

2.1 Electric theory

The type IIA supersymmetric electric brane configuration [10, 21, 22, 23] corresponding to $\mathcal{N} = 1$ $SU(N_c) \times SU(N'_c)$ gauge theory with N_f -fundamental flavors Q, \tilde{Q} , N'_f -fundamental flavors Q', \tilde{Q}' and bifundamentals X, \tilde{X} can be described as one middle NS5-brane, two NS5'-branes, N_c - and N'_c -D4-branes, and N_f - and N'_f -D6-branes. The X is in the representation (\square, \square) while the \tilde{X} is in the representation $(\bar{\square}, \bar{\square})$ under the gauge group. The quarks Q and \tilde{Q} are in the representation $(\square, \mathbf{1})$ and $(\bar{\square}, \mathbf{1})$ respectively under the gauge group. Similarly, the quarks Q' and \tilde{Q}' are in the representation $(\mathbf{1}, \square)$ and $(\mathbf{1}, \bar{\square})$ respectively under the gauge group. The mass terms for each fundamental quarks can be added by displacing each D6-branes along

$$v \equiv x^4 + ix^5$$

direction leading to their coordinates $v = +v_{D6_{-\theta}}(+v_{D6_{-\theta'}})$ respectively while the quartic terms for each fundamental quarks can be added also by rotating each D6-branes by an angle

$-\theta(-\theta')$ in (w, v) -plane respectively. Here we define the complex coordinate w as

$$w \equiv x^8 + ix^9.$$

Then, in the electric gauge theory, the general superpotential is given by

$$\begin{aligned} W_{elec} = & \frac{\alpha}{2} \text{tr}(Q\tilde{Q})^2 - m \text{tr} Q\tilde{Q} + \frac{\alpha'}{2} \text{tr}(Q'\tilde{Q}')^2 - m' \text{tr} Q'\tilde{Q}' \\ & + \left[-\frac{\beta}{2} \text{tr}(X\tilde{X})^2 + m_X \text{tr} X\tilde{X} \right] \end{aligned} \quad (2.1)$$

where $\alpha \equiv \frac{\tan \theta}{\Lambda}$ and $m \equiv \frac{v_{D6-\theta}}{2\pi\ell_s^2}$ for N_f D6-branes and similarly $\alpha' \equiv \frac{\tan \theta'}{\Lambda'}$ and $m' \equiv \frac{v_{D6-\theta'}}{2\pi\ell_s'^2}$ for N'_f D6-branes. The last two terms of (2.1) are due to the rotation angles ω_L and ω_R of two NS5'-branes in (w, v) -plane where $\beta \equiv (\tan \omega_L + \tan \omega_R)$ and the relative displacement of two color D4-branes where the mass for the bifundamentals $m_X \equiv \frac{v_{NS5'}}{2\pi\ell_s^2}$ is the distance of D4-branes along the v -direction. We focus on the particular limit $\beta, m_X \rightarrow 0$.

Then the $\mathcal{N} = 1$ supersymmetric electric brane configuration for the superpotential (2.1) in type IIA string theory is given as follows and let us draw this brane structure in Figure 1 explicitly:

- One middle NS5-brane in (012345) directions
- Two NS5'-branes in (012389) directions
- N_f $D6_{-\theta}$ -branes in (01237) directions and two other directions in (v, w) -plane
- N'_f $D6_{-\theta'}$ -branes in (01237) directions and two other directions in (v, w) -plane
- N_c - and N'_c -color D4-branes in (01236) directions

2.2 Magnetic theory

The left NS5'-brane starts out with linking number $l_e = -\frac{N'_f}{2} + N_c$ and after duality this left NS5'-brane ends up with linking number $l_m = \frac{N'_f}{2} - \tilde{N}'_c + N_f$. In general, when the N_f $D6_{-\theta}$ -branes meet the middle NS5-brane during the dual process, the new D4-branes are created because they are not parallel. However, we consider only the particular brane motion where N_f $D6_{-\theta}$ -branes meet the middle NS5-brane with no angles. In other words, the $D6_{-\theta}$ -branes become $D6_{-\frac{\pi}{2}}$ -branes when they meet with the middle NS5-brane instantaneously and then after that they come back to the original $D6_{-\theta}$ -branes. Therefore, in this dual process, there is no creation of D4-branes. That is the reason for the N_f factor in the l_m , not $2N_f$. Then it turns out that the dual color number \tilde{N}'_c is given by $\tilde{N}'_c = N_f + N'_f - N_c$.

What about the other dual color number? Note that we take the Seiberg dual for both gauge group factors. The right NS5'-brane starts out with linking number $l_e = \frac{N_f}{2} - N'_c$

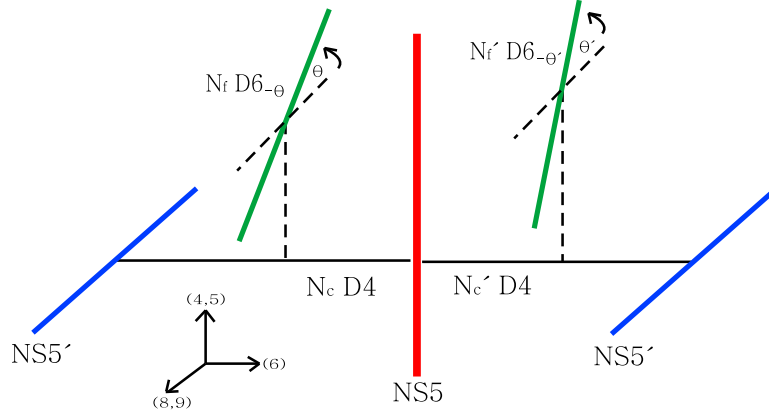


Figure 1: The $\mathcal{N} = 1$ supersymmetric electric brane configuration for the gauge group $SU(N_c) \times SU(N'_c)$ with bifundamentals X, \tilde{X} and fundamentals $Q, \tilde{Q}, Q', \tilde{Q}'$. A rotation of $N_f(N'_f)$ D6-branes in (w, v) -plane corresponds to a quartic term for the fundamentals $Q, \tilde{Q}(Q', \tilde{Q}')$ while a displacement of $N_f(N'_f)$ D6-branes in $+v$ direction corresponds to a mass term for the fundamentals $Q, \tilde{Q}(Q', \tilde{Q}')$.

and after duality this right NS5'-brane ends up with linking number $l_m = -\frac{N_f}{2} + \tilde{N}_c - N'_f$. In general, when the N'_f $D6_{-\theta'}$ -branes meet the middle NS5-brane during the dual process, the new D4-branes are created because they are not parallel. However, we consider only the particular brane motion where N'_f $D6_{-\theta'}$ -branes meet the middle NS5-brane with no angles. In other words, the $D6_{-\theta'}$ -branes become $D6_{-\frac{\pi}{2}}$ -branes when they meet with the middle NS5-brane instantaneously and after that they come back to the original $D6_{-\theta'}$ -branes. Therefore, in this dual process, there is no creation of D4-branes. That is the reason for the N'_f factor in the l_m , not $2N'_f$. Then it turns out that the dual color number \tilde{N}_c is given by $\tilde{N}_c = N'_f + N_f - N'_c$. Finally, one has the following dual color numbers

$$\tilde{N}_c = N'_f + N_f - N'_c, \quad \tilde{N}'_c = N_f + N'_f - N_c.$$

The low energy theory on the color D4-branes has $SU(\tilde{N}_c) \times SU(\tilde{N}'_c)$ gauge group and N_f -fundamental dual quarks q', \tilde{q}' coming from 4-4 strings connecting between the color \tilde{N}'_c D4-branes and N_f flavor D4-branes, N'_f -fundamental dual quarks q, \tilde{q} coming from 4-4 strings connecting between the color \tilde{N}_c D4-branes and N'_f flavor D4-branes as well as Y, \tilde{Y} and various gauge singlets. The Y is in the representation (\square, \square) while the \tilde{Y} is in the representation $(\bar{\square}, \bar{\square})$ under the dual gauge group. The N'_f flavors q and \tilde{q} are in the representation $(\square, \mathbf{1})$ and $(\bar{\square}, \mathbf{1})$ respectively under the gauge group and in the representation $(\bar{\square}, \mathbf{1})$ and $(\mathbf{1}, \bar{\square})$ respectively under the flavor group $SU(N'_f)_L \times SU(N'_f)_R$. Similarly, the N_f flavors q' and \tilde{q}' are in the representation $(\mathbf{1}, \square)$ and $(\mathbf{1}, \bar{\square})$ respectively under the gauge group and in the representation $(\bar{\square}, \mathbf{1})$ and $(\mathbf{1}, \bar{\square})$ respectively under the flavor group $SU(N_f)_L \times SU(N_f)_R$. In

particular, a magnetic meson field

$$M_0 \equiv Q\tilde{Q}$$

is $N_f \times N_f$ matrix and comes from 4-4 strings of N_f flavor D4-branes while a magnetic meson field

$$M'_0 \equiv Q'\tilde{Q}'$$

is $N'_f \times N'_f$ matrix and comes from 4-4 strings of N'_f flavor D4-branes. Then the most general magnetic superpotential is given by

$$\begin{aligned} W_{dual} = & \left[(Y\tilde{Y})^2 + Y\tilde{Y} + M_0 q' \tilde{Y} Y \tilde{q}' + M'_0 \tilde{q} Y \tilde{Y} q + M_1 q' \tilde{q}' + M'_1 \tilde{q} q + P q \tilde{Y} q' + \tilde{P} \tilde{q}' Y \tilde{q} \right] \\ & + \frac{\alpha}{2} \text{tr} M_0^2 - m M_0 + \frac{\alpha'}{2} \text{tr} M'^2_0 - m' M'_0 \end{aligned} \quad (2.2)$$

where the mesons are $M_1 \equiv Q\tilde{X}X\tilde{Q}$, $M'_1 \equiv Q'\tilde{X}X\tilde{Q}'$, $P \equiv Q\tilde{X}Q'$ and $\tilde{P} \equiv \tilde{Q}X\tilde{Q}'$. The expression in the first line of (2.2) was found in [14, 15] in the gauge theory side already. Note that the mesons of the $SU(N_c)$ group couple to the dual quarks of $SU(\tilde{N}'_c)$ and the mesons of the $SU(N'_c)$ group couple to the dual quarks of $SU(\tilde{N}_c)$ ¹.

As we explained before, our particular brane motion during the dual process does not produce any D4-branes when the N_f $D6_{-\theta}$ -branes meet the middle NS5-brane. This implies that there is no M_1 term in the above superpotential (2.2). The meson M_1 originates from $SU(N_c)$ chiral mesons $Q\tilde{Q}$ when one dualizes the first gauge group factor first by moving the middle NS5-brane to the left of the left NS5'-brane. That is, the fluctuations of strings stretching between the N_f “flavor” D4-branes correspond to this meson field. The superpotential contains the cubic term between this meson field and dual quarks. After two additional dual procedures, $SU(N'_c)$ and $SU(\tilde{n}_c)$, this cubic term arises as M_1 -term in (2.2) where M_1 has an extra $\tilde{X}X$ fields besides $Q\tilde{Q}$, due to the further $SU(N'_c)$ -dualization.

Similarly, because there is no creation of D4-branes when the N'_f $D6_{-\theta'}$ -branes meet the middle NS5-brane, there is no M'_1 term in the above superpotential (2.2) also. The meson M'_1

¹As suggested in [10], one can dualize each gauge group independently of the other. For example, one dualizes the second gauge group factor by moving the middle NS5-brane to the right of the right NS5'-brane, like as in [23]. Then the dual gauge group is given by $SU(N_c) \times SU(\tilde{n}'_c = N'_f + N_c - N'_c)$ with corresponding superpotential for the dual matters. Now we interchange the NS5'-branes and $D6_{-\theta, -\theta'}$ -branes each other and obtain the next dual gauge group $SU(\tilde{n}_c = 2N'_f + N_f - N'_c) \times SU(\tilde{n}'_c)$ with dual matters. Finally, we move the left NS5'-brane and $D6_{-\theta}$ -branes (in the electric theory) to the right of the middle NS5-brane and obtain the final dual gauge group $SU(\tilde{N}_c = \tilde{n}_c) \times SU(\tilde{N}'_c = 2N_f + N'_f - N_c)$ with the superpotential (2.2). We also obtain the same dual gauge theory if we start with $SU(N_c)$ dualization first and then $SU(N'_c)$ dualization and finally end up with $SU(n_c)$ dualization.

originates from $SU(N'_c)$ chiral mesons $Q'\tilde{Q}'$ when one dualizes the second gauge group factor first by moving the middle NS5-brane to the right of the right NS5'-brane. That is, the strings stretching between the N'_f “flavor” D4-branes provides this meson. The superpotential in [23] contains the cubic term between this meson field and dual quarks. After two additional dual procedures, $SU(N_c)$ and $SU(\tilde{n}'_c)$ observed in the footnote 1, this cubic term arises as M'_1 -term in (2.2) where M'_1 has extra $\tilde{X}X$ fields besides $Q'\tilde{Q}'$, due to the further $SU(N_c)$ -dualization.

Furthermore, we do not see any P - or \tilde{P} -dependent terms in the superpotential (2.2) when the N_f $D6_{-\theta}$ -branes, the N_f $D6_{-\theta'}$ -branes and the middle NS5-brane during the dual process meet each other with no angles. These mesons P and \tilde{P} originate from $SU(N'_c)$ chiral mesons $\tilde{X}Q'$ and $X\tilde{Q}'$ [23] when one dualizes the second gauge group factor first by moving the middle NS5-brane to the right of the right NS5'-brane, as in footnote 1. That is, the strings stretching between the N'_f flavor D4-branes and N_c color D4-branes give rise to these N'_f $SU(N_c)$ fundamentals and N'_f $SU(N_c)$ antifundamentals. The superpotential in [23] contains the cubic term between dual bifundamental, these meson fields and dual quarks. After two additional dual procedures, $SU(N_c)$ and $SU(\tilde{n}'_c)$, these cubic terms arise as P and \tilde{P} -term in (2.2) where there exist extra quarks q and \tilde{q} while P and \tilde{P} have extra Q and \tilde{Q} fields, due to the further $SU(N_c)$ -dualization.

All these features for selecting the wanted gauge singlet during the dual process has occurred also in the different gauge theories [24, 25] where the gauge group is a single gauge group with the presence of O6-plane and in these cases the brane does not move independently due to the O6-plane. Then the reduced magnetic superpotential in our case with the limit $\beta, m_X \rightarrow 0$ is given by

$$W_{dual} = \left[M_0 q' \tilde{Y} Y \tilde{q}' + \frac{\alpha}{2} \text{tr} M_0^2 - m M_0 \right] + \left[M'_0 \tilde{q} Y \tilde{Y} q + \frac{\alpha'}{2} \text{tr} M'^2_0 - m' M'_0 \right]. \quad (2.3)$$

For the supersymmetric vacua, one can compute the F-term equations for this superpotential (2.3) and the expectation values for $M_0, M'_0, q' \tilde{Y} Y \tilde{q}'$ and $\tilde{q} Y \tilde{Y} q$ are obtained. The F-term equations for $M_0, q', \tilde{q}', M'_0, q, \tilde{q}, Y$ and \tilde{Y} are

$$\begin{aligned} q' \tilde{Y} Y \tilde{q}' - m + \alpha M_0 &= 0, & \tilde{Y} (Y \tilde{q}' M_0) &= 0, & (M_0 q' \tilde{Y}) Y &= 0, \\ \tilde{q} Y \tilde{Y} q - m' + \alpha' M'_0 &= 0, & (M'_0 \tilde{q} Y) \tilde{Y} &= 0, & Y (\tilde{Y} q M'_0) &= 0, \\ \tilde{q}' (M_0 q' \tilde{Y}) + (\tilde{Y} q M'_0) \tilde{q} &= 0, & (Y \tilde{q}' M_0) q' + q (M'_0 \tilde{q} Y) &= 0. \end{aligned} \quad (2.4)$$

The seventh and eighth equations of (2.4) are satisfied if the the second, third, fifth and sixth equations of (2.4) hold: $Y \tilde{q}' M_0 = M_0 q' \tilde{Y} = M'_0 \tilde{q} Y = \tilde{Y} q M'_0 = 0$. We present the magnetic brane configuration in Figure 2. Depending on the values of the masses m and m' , there exist

other three possibilities: the v coordinates of $D6_{-\theta}$ -branes and $D6_{-\theta'}$ -branes are classified as $v = (+, -), (-, +)$ and $(-, -)$. Of course, if we put an orientifold 6-plane at the origin, then this figure reduces to the one in [24] where the matter contents and superpotential should be preserved under the O6-plane.

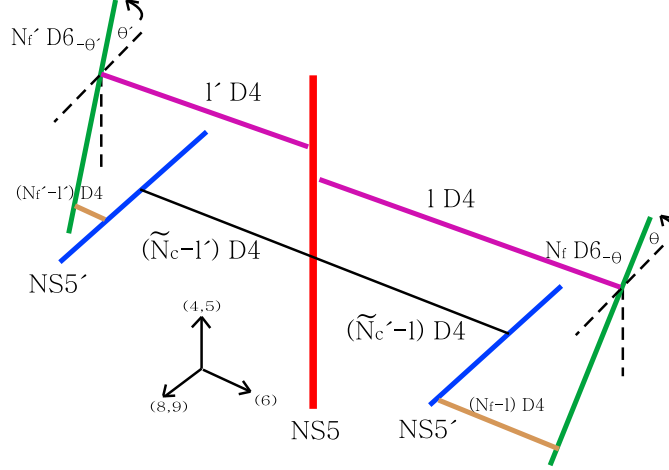


Figure 2: The $\mathcal{N} = 1$ supersymmetric magnetic brane configuration corresponding to Figure 1 with a splitting and a reconnection between D4-branes when the gravitational potential of the NS5-brane is ignored. The N_f flavor D4-branes connecting between $D6_{-\theta}$ -branes and NS5'-brane are splitting into $(N_f - l)$ - and l - D4-branes while the N'_f flavor D4-branes connecting between $D6_{-\theta'}$ -branes and NS5'-brane are splitting into $(N'_f - l')$ - and l' - D4-branes. The v and w coordinates of each these D4-branes are related to each other through the deformation parameters θ and θ' respectively.

The theory has many nonsupersymmetric meta-stable ground states and when we rescale the meson fields as

$$M_0 = h\Lambda\Phi_0 \quad \text{and} \quad M'_0 = h'\Lambda'\Phi'_0,$$

then the Kahler potential for Φ_0 and Φ'_0 is canonical and the magnetic quarks are canonical near the origin of field space [1]. Then the magnetic superpotential (2.3) can be rewritten as

$$W_{mag} = \left[h\Phi_0 q' \tilde{Y} Y \tilde{q}' + \frac{h^2 \mu_\phi}{2} \text{tr} \Phi_0^2 - h\mu^2 \text{tr} \Phi_0 \right] + \left[h'\Phi'_0 \tilde{q} Y \tilde{Y} q + \frac{h'^2 \mu'_\phi}{2} \text{tr} \Phi_0'^2 - h'\mu'^2 \text{tr} \Phi'_0 \right]$$

where $\mu^2 = m\Lambda, \mu'^2 = m'\Lambda'$ and $\mu_\phi = \alpha\Lambda^2, \mu'_\phi = \alpha'\Lambda'^2$.

Now one splits the $(N_f - l) \times (N_f - l)$ block at the lower right corner of $h\Phi_0$ and $q' \tilde{Y} Y \tilde{q}'$ into blocks of size n and $(N_f - l - n)$ and one decomposes the $(N'_f - l') \times (N'_f - l')$ block at

the lower right corner of $h'\Phi'_0$ and $\tilde{q}Y\tilde{Y}q$ into blocks of size n' and $(N'_f - l' - n')$ as follows [3]:

$$\begin{aligned} h\Phi_0 &= \begin{pmatrix} 0_l & 0 & 0 \\ 0 & h\Phi_n & 0 \\ 0 & 0 & \frac{\mu^2}{\mu_\phi} \mathbf{1}_{N_f-l-n} \end{pmatrix}, & h'\Phi'_0 &= \begin{pmatrix} 0_{l'} & 0 & 0 \\ 0 & h'\Phi'_{n'} & 0 \\ 0 & 0 & \frac{\mu'^2}{\mu'_\phi} \mathbf{1}_{N'_f-l'-n'} \end{pmatrix}, \\ q'\tilde{Y}Y\tilde{q}' &= \begin{pmatrix} \mu^2 \mathbf{1}_l & 0 & 0 \\ 0 & \varphi' \tilde{y} y \tilde{\varphi}' & 0 \\ 0 & 0 & 0_{N_f-l-n} \end{pmatrix}, & \tilde{q}Y\tilde{Y}q &= \begin{pmatrix} \mu'^2 \mathbf{1}_{l'} & 0 & 0 \\ 0 & \tilde{\varphi} y \tilde{y} \varphi & 0 \\ 0 & 0 & 0_{N'_f-l'-n'} \end{pmatrix}. \end{aligned}$$

Here φ' and $\tilde{\varphi}'$ are $n \times (\tilde{N}'_c - l)$ dimensional matrices and correspond to n flavors of fundamentals of the gauge group $SU(\tilde{N}'_c - l)$ which is unbroken and φ and $\tilde{\varphi}$ are $n' \times (\tilde{N}_c - l')$ dimensional matrices and correspond to n' flavors of fundamentals of the gauge group $SU(\tilde{N}_c - l')$ which is unbroken. In the brane configuration shown in Figure 3, φ' and $\tilde{\varphi}'$ correspond to fundamental strings connecting the n flavor D4-branes and $(\tilde{N}'_c - l)$ color D4-branes and φ and $\tilde{\varphi}$ correspond to fundamental strings connecting the n' flavor D4-branes and $(\tilde{N}_c - l')$ color D4-branes. The Φ_n and $\varphi' \tilde{y} y \tilde{\varphi}'$ are $n \times n$ matrices while $\Phi'_{n'}$ and $\tilde{\varphi} y \tilde{y} \varphi$ are $n' \times n'$ matrices. The supersymmetric ground state corresponds to

$$h\Phi_n = \frac{\mu^2}{\mu_\phi} \mathbf{1}_n, \quad \varphi' \tilde{y} = 0 = y \tilde{\varphi}' \quad \text{and} \quad h'\Phi'_{n'} = \frac{\mu'^2}{\mu'_\phi} \mathbf{1}_{n'}, \quad \tilde{y} \varphi = 0 = \tilde{\varphi} y.$$

Now the full one loop potential, by combining the superpotential and the vacuum expectation values for the fields, takes the form

$$\begin{aligned} V &= |h\Phi_n \varphi' \tilde{y}|^2 + |h y \tilde{\varphi}' \Phi_n|^2 + |h \varphi' \tilde{y} y \tilde{\varphi}' - h \mu^2 \mathbf{1}_n + h^2 \mu_\phi \Phi_n|^2 + b |h^2 \mu|^2 \text{tr} \Phi_n^\dagger \Phi_n \\ &+ |h' \Phi'_{n'} \tilde{\varphi} y|^2 + |h' \tilde{y} \varphi \Phi'_{n'}|^2 + |h' \tilde{\varphi} y \tilde{y} \varphi - h' \mu'^2 \mathbf{1}_{n'} + h'^2 \mu'_\phi \Phi'_{n'}|^2 + b' |h'^2 \mu'|^2 \text{tr} \Phi'^{\dagger}_{n'} \Phi'_{n'}, \end{aligned}$$

where $b = \frac{(\ln 4 - 1)}{8\pi^2} \tilde{N}_c$ and $b' = \frac{(\ln 4 - 1)}{8\pi^2} \tilde{N}'_c$ [1]. Differentiating this potential with respect to Φ_n^\dagger and $\Phi'^{\dagger}_{n'}$ and putting $\varphi' \tilde{y} = 0 = y \tilde{\varphi}'$ and $\tilde{\varphi} y = 0 = \tilde{y} \varphi$ [3], one obtains, using the methods given in [16, 17, 18, 19, 20],

$$\begin{aligned} h\Phi_n &\simeq \frac{\mu_\phi}{b} \mathbf{1}_n \quad \text{or} \quad M_n \simeq \frac{\alpha \Lambda^3}{\tilde{N}_c} \mathbf{1}_n, \\ h'\Phi'_{n'} &\simeq \frac{\mu'_\phi}{b'} \mathbf{1}_{n'} \quad \text{or} \quad M'_{n'} \simeq \frac{\alpha' \Lambda'^3}{\tilde{N}'_c} \mathbf{1}_{n'} \end{aligned}$$

corresponding to the w coordinates of n curved flavor D4-branes between the $D6_{-\theta}$ -branes and the NS5'-brane and the w coordinates of n' curved flavor D4-branes between the $D6_{-\theta'}$ -branes and the NS5'-brane respectively.

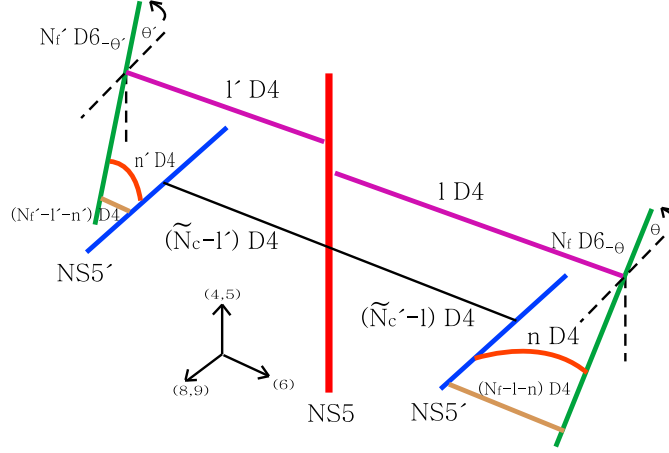


Figure 3: The nonsupersymmetric meta-stable magnetic brane configuration corresponding to Figure 1 with a misalignment between D4-branes when the gravitational potential of the NS5-brane is considered. The $(N_f - l)$ flavor D4-branes in Figure 2 connecting between $D6_{-\theta}$ -branes and NS5'-brane are further splitting into $(N_f - l - n)$ - and n -curved D4-branes while the $(N_f' - l')$ flavor D4-branes in Figure 2 connecting between $D6_{-\theta'}$ -branes and NS5'-brane are further splitting into $(N_f' - l' - n')$ - and n' -curved D4-branes. When there are two multiple NS5'-branes, further distributions of D4-branes arise along the v direction.

2.3 Higher order superpotential for bifundamentals

In general, there are also different meson fields $M_j = Q(\tilde{X}X)^j\tilde{Q}$, $M'_j = Q'(\tilde{X}X)^j\tilde{Q}'$, $P_r = Q(\tilde{X}X)^{r-1}\tilde{X}Q'$ and $\tilde{P}_r = \tilde{Q}X(\tilde{X}X)^{r-1}\tilde{Q}'$ where $j = 0, 1, \dots, k' - 1$ and $r = 1, \dots, k'$ for higher order superpotential for bifundamental with general rotation angles of two k' NS5'-branes [14, 15, 10]. The magnetic superpotential contains the interaction between these meson fields with $q, \tilde{q}, q', \tilde{q}', Y$ and \tilde{Y} [14] as well as the higher order term for dual bifundamental $(Y\tilde{Y})^{k'+1}$ and mass term for dual bifundamental $Y\tilde{Y}$ which will vanish for small β, m_X limit. When the N_f $D6_{-\theta}$ -branes and the extra right $(k' - 1)$ NS5'-branes do not create the D4-branes and N_f' $D6_{-\theta'}$ -branes and the extra left $(k' - 1)$ NS5'-branes also do not create the D4-branes, the extra meson fields M_j, M'_j, P_r and \tilde{P}_r where $j \neq 0$ do not appear in the magnetic superpotential.

Then the k' -dependent magnetic superpotential with the limit $\beta, m_X \rightarrow 0$ is given by

$$W_{dual} = \left[M_0 q' (\tilde{Y}Y)^{k'} \tilde{q}' + \frac{\alpha}{2} \text{tr} M_0^2 - m M_0 \right] + \left[M'_0 \tilde{q} (Y\tilde{Y})^{k'} q + \frac{\alpha'}{2} \text{tr} M_0'^2 - m' M'_0 \right].$$

Then the analysis for the previous single NS5'-brane case can be performed in this case also. One deforms the generalized Figure 3, where there are multiple NS5'-branes, by displacing the multiple $D6_{-\theta, -\theta'}$ -branes and NS5'-branes along v direction, as in [19, 20]. Then the n curved flavor D4-branes attached to them (as well as other D4-branes) are displaced also as

k' different n_j 's connecting between $D6_{-\theta,j}$ -brane and the right $NS5'_j$ -brane($j = 1, 2, \dots, k'$). Similarly, the n' curved flavor D4-branes attached to them(as well as other D4-branes) are displaced also as k' different n'_j 's connecting between $D6_{-\theta',j}$ -brane and the left $NS5'_j$ -brane. One can consider the particular case $N_f = k' = N'_f$. When we rescale the submeson fields as $M_j = h\Lambda\Phi_j$ and $M'_j = h'\Lambda'\Phi'_j$ [19], then the Kahler potential for Φ_j and Φ'_j is canonical and the magnetic quarks q_j, \tilde{q}_j and q'_j, \tilde{q}'_j are canonical near the origin of field space [1]. Then the magnetic superpotential can be rewritten in terms of $\Phi_j, q_j, \tilde{q}_j, \Phi'_j, q'_j, \tilde{q}'_j, Y_j, Y_{j'}, \tilde{Y}_j$ and $\tilde{Y}_{j'}$. In order to see the nonsupersymmetric meta-stable ground states, one can split some of the components of $h\Phi_0, q'(\tilde{Y}Y)^{k'}\tilde{q}', h'\Phi'_0$ and $\tilde{q}(Y\tilde{Y})^{k'}q$ as usual. The supersymmetric ground state corresponds to

$$\begin{aligned} h\Phi_{n_j} &= \frac{\mu_j^2}{\mu_\phi} \mathbf{1}_{n_j}, & \varphi'_{n_j} (\tilde{y}_{n_j} y_{n_j})^{\frac{k'}{2}} = 0 = (\tilde{y}_{n_j} y_{n_j})^{\frac{k'}{2}} \tilde{\varphi}'_{n_j} & \quad \text{and} \\ h'\Phi'_{n'_j} &= \frac{\mu_j'^2}{\mu_\phi'} \mathbf{1}_{n'_j}, & (y_{n'_j} \tilde{y}_{n'_j})^{\frac{k'}{2}} \varphi_{n'_j} = 0 = \tilde{\varphi}_{n'_j} (y_{n'_j} \tilde{y}_{n'_j})^{\frac{k'}{2}} \end{aligned}$$

when k' is even. For k' odd, one gets similar supersymmetric ground states. The full one loop potential can be written similarly and the local nonzero stable point arises as

$$h\Phi_{n_j} \simeq \frac{\mu_\phi}{b_j} \mathbf{1}_{n_j} \quad \text{and} \quad h'\Phi'_{n'_j} \simeq \frac{\mu_\phi'}{b'_j} \mathbf{1}_{n'_j}$$

corresponding to the w coordinates of n_j curved flavor D4-branes between the $D6_{-\theta,j}$ -branes and the $NS5'_{R,j}$ -brane and the w coordinates of n'_j curved flavor D4-branes between the $D6_{-\theta',j}$ -branes and the $NS5'_{L,j}$ -brane respectively.

Therefore, the meta-stable states, for fixed k' which is related to the order of the bifundamental field in the superpotential and θ and θ' which are deformation parameters by rotation angles of $D6_{-\theta,-\theta'}$ -branes, are classified by the number of various D4-branes and the positions of multiple $D6_{-\theta,-\theta'}$ -branes and NS5'-branes [19, 20]:

$$(N_{c,j}, N_{f,j}, l_j, n_j, N'_{c,j}, N'_{f,j}, l'_j, n'_j) \quad \text{and} \quad (v_{D6_{-\theta,j}}, v_{D6_{-\theta',j}}, v_{NS5'_{L,j}}, v_{NS5'_{R,j}})$$

where $j = 1, 2, \dots, k'$. The description of other range for the N_f, N'_f and k' can be analyzed similarly.

3 $SO(2N_c) \times Sp(N'_c)$ with $2N_f$ -vectors, $2N'_f$ -fund. and bifund.

Let us add orientifold 4-plane to the previous brane configuration.

3.1 Electric theory

The type IIA supersymmetric electric brane configuration [26, 11, 12] corresponding to $\mathcal{N} = 1$ $SO(2N_c) \times Sp(N'_c)$ gauge theory with $2N_f$ -vectors Q , N'_f -fundamental flavors Q' and bifundamental X can be described as one middle NS5-brane, two NS5'-branes, $2N_c$ - and $2N'_c$ -D4-branes, and $2N_f$ - and $2N'_f$ -D6-branes and $O4^\pm$ -planes. The mass terms can be achieved by displacing each D6-branes along $\pm v$ direction leading to their coordinates $v = \pm v_{D6-\theta}(\pm v_{D6-\theta'})$ respectively while the quartic terms for the quarks can be obtained by rotating the D6-branes by an angle $-\theta(-\theta')$ in (w, v) -plane respectively. Then, in the electric gauge theory, the general superpotential is given by

$$W_{elec} = \frac{\alpha}{2} \text{tr}(QQ)^2 - m \text{tr} QQ + \frac{\alpha'}{2} \text{tr}(Q'Q')^2 - m' \text{tr} Q'Q' + \left[-\frac{\beta}{2} \text{tr}(XX)^2 + m_X \text{tr} XX \right]. \quad (3.1)$$

The last two terms of (3.1) are due to the rotation angle ω of NS5'-branes where $\beta = \tan \omega$ and the relative displacement of D4-branes where the mass for the bifundamental $m_X = \frac{v_{NS5'}}{2\pi \ell_s^2}$ is the distance of D4-branes in v direction. We focus on the limit $\beta, m_X \rightarrow 0$.

Let us summarize the $\mathcal{N} = 1$ supersymmetric electric brane configuration for the superpotential (3.1) in type IIA string theory and we do not draw this here but it is given by Figure 1 with the appearance of mirrors for an orientifold 4-plane.

- One middle NS5-brane in (012345) directions
- Two NS5'-branes in (012389) directions
- $2N_f$ $D6_{-\theta}$ -branes in (01237) directions and two other directions in (v, w) -plane
- $2N'_f$ $D6_{-\theta'}$ -branes in (01237) directions and two other directions in (v, w) -plane
- $2N_c$ - and $2N'_c$ -color D4-branes in (01236) directions
- $O4^\pm$ -planes in (01236) directions

3.2 Magnetic theory

The left NS5'-brane starts out with linking number $l_e = -\frac{(2N'_f)}{2} - 2 + 2N_c$ and after duality this left NS5'-brane ends up with linking number $l_m = \frac{(2N'_f)}{2} - 2 - 2\tilde{N}'_c + 2N_f$. We consider only the particular brane motion where $2N_f$ $D6_{-\theta}$ -branes meet the middle NS5-brane with no angles. In other words, the $D6_{-\theta}$ -branes become $D6_{-\frac{\pi}{2}}$ -branes when they meet with NS5-brane instantaneously and then after that they come back to the original $D6_{-\theta}$ -branes. Then it turns out that the dual color number $2\tilde{N}'_c$ is given by $2\tilde{N}'_c = 2N_f + 2N'_f - 2N_c$.

The right NS5'-brane starts out with linking number $l_e = \frac{(2N_f)}{2} - 2 - 2N'_c$ and after duality this right NS5'-brane ends up with linking number $l_m = -\frac{(2N_f)}{2} - 2 + 2\tilde{N}_c - 2N'_f$.

We consider only the particular brane motion where $2N'_f$ $D6_{-\theta'}$ -branes meet the middle NS5-brane with no angles. The $D6_{-\theta'}$ -branes become $D6_{-\frac{\pi}{2}}$ -branes when they meet with NS5-brane instantaneously and then after that they come back to the original $D6_{-\theta'}$ -branes. Then it turns out that the dual color number $2\tilde{N}_c$ is given by $2\tilde{N}_c = 2N'_f + 2N_f - 2N'_c$. Finally, one has the following dual color numbers

$$2\tilde{N}_c = 2N'_f + 2N_f - 2N'_c, \quad 2\tilde{N}'_c = 2N_f + 2N'_f - 2N_c.$$

The low energy theory on the color D4-branes has $SO(2\tilde{N}_c) \times Sp(\tilde{N}'_c)$ gauge group and $2N_f$ -fundamental dual quarks q' coming from 4-4 strings connecting between the color $2\tilde{N}'_c$ D4-branes and $2N_f$ flavor D4-branes, $2N'_f$ -fundamental dual quarks q coming from 4-4 strings connecting between the color $2\tilde{N}_c$ D4-branes and $2N'_f$ flavor D4-branes as well as Y and various gauge singlets. The $2N'_f$ flavors q and $2N_f$ flavors q' are in the representation $(\square, \mathbf{1})$ and $(\mathbf{1}, \square)$ respectively under the gauge group and in the representation $(\mathbf{1}, \bar{\square})$ and $(\bar{\square}, \mathbf{1})$ respectively under the flavor group $SU(2N_f) \times SU(2N'_f)$. In particular, a magnetic meson field

$$M_0 \equiv QQ$$

is $2N_f \times 2N_f$ symmetric matrix and comes from 4-4 strings of $2N_f$ flavor D4-branes while a magnetic meson field

$$M'_0 \equiv Q'Q'$$

is $2N'_f \times 2N'_f$ antisymmetric matrix and comes from 4-4 strings of $2N'_f$ flavor D4-branes. Then the most general magnetic superpotential is given by

$$\begin{aligned} W_{dual} &= [(YY)^2 + YY + M_0 q' Y Y q' + M'_0 q Y Y q + M_1 q' q' + M'_1 q q + P q Y q'] \\ &+ \frac{\alpha}{2} \text{tr} M_0^2 - m M_0 + \frac{\alpha'}{2} \text{tr} M'^2_0 - m' M'_0 \end{aligned} \quad (3.2)$$

where the mesons are $M_1 \equiv Q X X Q$, $M'_1 \equiv Q' X X Q'$ and $P \equiv Q X Q'$. The first line of (3.2) was appeared in [14]. Note that the mesons of the $SO(2N_c)$ group couple to the dual quarks of $Sp(\tilde{N}'_c)$ and the mesons of the $Sp(N'_c)$ group couple to the dual quarks of $SO(2\tilde{N}_c)$.

One can dualize each gauge group independently of the other. For example, one dualizes the second gauge group factor by moving the middle NS5-brane to the right of the right NS5'-brane, like as in [26]. Then the dual gauge group is given by $SO(2N_c) \times Sp(\tilde{n}'_c = N'_f + N_c - N'_c - 2)$ with corresponding superpotential for the dual matters. Now we interchange the NS5'-branes and $D6_{-\theta, -\theta'}$ -branes each other and obtain the dual gauge group $SO(2\tilde{n}_c = 4N'_f + 2N_f - 2N'_c) \times Sp(\tilde{n}'_c)$ with dual matters. Finally, we move the left NS5'-brane and

$D6_{-\theta}$ -branes in the electric theory to the right of the middle NS5-brane and obtain the final dual gauge group $SO(2\tilde{N}_c = 2\tilde{n}_c) \times Sp(\tilde{N}'_c = 2N_f + N'_f - N_c)$ with the superpotential (3.2) ².

As before, our particular brane motion during the dual process does not produce any D4-branes when the $2N_f$ $D6_{-\theta}$ -branes meet the middle NS5-brane. This implies that there is no M_1 term in the above superpotential (3.2) ³. Similarly, when the $2N'_f$ $D6_{-\theta'}$ -branes meet the middle NS5-brane, the fact that there is no creation of D4-branes leads to the fact that there is no M'_1 term in the above superpotential (3.2) also ⁴. Furthermore, we do not see any P -dependent term in the superpotential (3.2) when the $2N_f$ $D6_{-\theta}$ -branes, the $2N'_f$ $D6_{-\theta'}$ -branes and the middle NS5-brane, during the dual process, meet each other with no angles ⁵. Then the reduced magnetic superpotential in our case with the limit $\beta, m_X \rightarrow 0$ is given by

$$W_{dual} = \left[M_0 q' Y Y q' + \frac{\alpha}{2} \text{tr} M_0^2 - m M_0 \right] + \left[M'_0 q Y Y q + \frac{\alpha'}{2} \text{tr} M'^2_0 - m' M'_0 \right]. \quad (3.3)$$

For the supersymmetric vacua, one can compute the F-term equations for this superpotential (3.3) and the expectation values for $M_0, M'_0, q' Y Y q'$ and $q Y Y q$ are obtained. The F-term equations for M_0, q', M'_0, q and Y are

$$\begin{aligned} q' Y Y q' - m + \alpha M_0 &= 0, & Y(Y q' M_0) &= 0, \\ q Y Y q - m' + \alpha' M'_0 &= 0, & (M'_0 q Y) Y &= 0, \\ (Y q' M_0) q' + q(M'_0 q Y) &= 0. \end{aligned} \quad (3.4)$$

The fifth equation of (3.4) is satisfied if the second and fourth equations of (3.4) hold: $Y q' M_0 =$

²As in unitary case of previous section, it does not matter the order of dualization. We obtain the same dual gauge theory if we start with $SO(2N_c)$ dualization, then $Sp(N'_c)$ dualization and end up with $SO(2\tilde{n}_c)$ dualization.

³The meson M_1 originates from $SO(2N_c)$ chiral mesons QQ when one dualizes the first gauge group factor first. That is, the fluctuation of the strings stretching between the $2N_f$ “flavor” D4-branes provides this meson field. The superpotential at this stage contains the cubic term between this meson field and dual quarks. After two additional dual procedures, $Sp(N'_c)$ and $SO(2\tilde{n}_c)$, this cubic term arises as M_1 -term in (3.2) where M_1 has an extra XX field-dependent factor besides QQ , due to the further $Sp(N'_c)$ -dualization.

⁴The meson M'_1 originates from $Sp(N'_c)$ chiral mesons $Q'Q'$ when one dualizes the second gauge group factor first. Then the fluctuations of the strings stretching between the $2N'_f$ “flavor” D4-branes gives this meson field. The superpotential in [26] contains the cubic term between this meson field and dual quarks. After two additional dual procedures, $SO(2N_c)$ and $Sp(\tilde{n}'_c)$, this cubic term arises as M'_1 -term in (3.2) where M'_1 has extra XX fields besides $Q'Q'$, due to the further $SO(2N_c)$ -dualization.

⁵The meson P originates from $Sp(N'_c)$ chiral meson XQ' [26] when one dualizes the second gauge group factor first. That is, the strings stretching between the $2N'_f$ “flavor” D4-branes and $2N_c$ color D4-branes give rise to these $2N'_f$ $SO(2N_c)$ flavors. The superpotential contains the cubic term between dual bifundamental, this meson field and dual quarks. After two additional dual procedures, $SO(2N_c)$ and $Sp(\tilde{n}'_c)$, this cubic term arises as P -term in (3.2) where there exist extra quarks q while P has an extra Q field, due to the further $SO(2N_c)$ -dualization.

$M'_0 qY = 0$. One can read off the corresponding magnetic brane configuration by adding O4-plane and mirrors from the Figure 2.

The theory has many nonsupersymmetric meta-stable ground states by requiring the IR free region for each gauge group factors [27] and when we rescale the meson fields as $M_0 = h\Lambda\Phi_0$ and $M'_0 = h'\Lambda'\Phi'_0$, then the Kahler potential for Φ_0 and Φ'_0 is canonical and the magnetic quarks are canonical near the origin of field space. Then the magnetic superpotential can be rewritten as

$$W_{mag} = \left[h\Phi_0 q'YYq' + \frac{h^2\mu_\phi}{2} \text{tr } \Phi_0^2 - h\mu^2 \text{tr } \Phi_0 \right] + \left[h'\Phi'_0 qYYq + \frac{h'^2\mu'_\phi}{2} \text{tr } \Phi'^2_0 - h'\mu'^2 \text{tr } \Phi'_0 \right]$$

where $\mu^2 = m\Lambda$, $\mu'^2 = m'\Lambda'$ and $\mu_\phi = \alpha\Lambda^2$, $\mu'_\phi = \alpha'\Lambda'^2$.

Now one splits the $2(N_f - l) \times 2(N_f - l)$ block at the lower right corner of $h\Phi_0$ and $q'YYq'$ into blocks of size $2n$ and $2(N_f - l - n)$ and one decomposes the $2(N'_f - l') \times 2(N'_f - l')$ block at the lower right corner of $h'\Phi'_0$ and $qYYq$ into blocks of size $2n'$ and $2(N'_f - l' - n')$ as follows [3]:

$$h\Phi_0 = \begin{pmatrix} 0_{2l} & 0 & 0 \\ 0 & h\Phi_{2n} & 0 \\ 0 & 0 & \frac{\mu^2}{\mu_\phi} \mathbf{1}_{N_f-l-n} \otimes \sigma_3 \end{pmatrix}, \quad h'\Phi'_0 = \begin{pmatrix} 0_{2l'} & 0 & 0 \\ 0 & h'\Phi'_{2n'} & 0 \\ 0 & 0 & \frac{\mu'^2}{\mu'_\phi} \mathbf{1}_{N'_f-l'-n'} \otimes i\sigma_2 \end{pmatrix},$$

$$q'YYq' = \begin{pmatrix} \mu^2 \mathbf{1}_{2l} & 0 & 0 \\ 0 & \varphi'yy\varphi' & 0 \\ 0 & 0 & 0_{2(N_f-l-n)} \end{pmatrix}, \quad qYYq = \begin{pmatrix} \mu'^2 \mathbf{1}_{2l'} & 0 & 0 \\ 0 & \varphi yy\varphi & 0 \\ 0 & 0 & 0_{2(N'_f-l'-n')} \end{pmatrix}.$$

Here φ' is $2n \times 2(\tilde{N}'_c - l)$ dimensional matrices and correspond to $2n$ flavors of the gauge group $Sp(\tilde{N}'_c - l)$ which is unbroken and φ is $2n' \times (2\tilde{N}_c - 2l')$ dimensional matrices and correspond to $2n'$ flavors of the gauge group $SO(2\tilde{N}_c - 2l')$ which is unbroken. The φ' corresponds to fundamental strings connecting the $2n$ flavor D4-branes and $2(\tilde{N}'_c - l)$ color D4-branes and φ corresponds to fundamental strings connecting the $2n'$ flavor D4-branes and $2(\tilde{N}_c - l')$ color D4-branes. The Φ_{2n} and $\varphi'yy\varphi'$ are $2n \times 2n$ matrices while $\Phi'_{2n'}$ and $\varphi yy\varphi$ are $2n' \times 2n'$ matrices. The supersymmetric ground state corresponds to

$$h\Phi_{2n} = \frac{\mu^2}{\mu_\phi} \mathbf{1}_n \otimes \sigma_3, \quad \varphi'y = 0 = y\varphi' \quad \text{and} \quad h'\Phi'_{2n'} = \frac{\mu'^2}{\mu'_\phi} \mathbf{1}_{n'} \otimes i\sigma_2, \quad y\varphi = 0 = \varphi y.$$

Now the full one loop potential from the above superpotential and vacuum expectation values for the fields takes the form

$$V = |hy\varphi'\Phi_{2n}|^2 + |h\varphi'yy\varphi' - h\mu^2 \mathbf{1}_{2n} + h^2\mu_\phi\Phi_{2n}|^2 + b|h^2\mu|^2 \text{tr } \Phi_{2n}\Phi_{2n} \\ + |h'\Phi'_{2n'}\varphi y|^2 + |h'\varphi yy\varphi - h'\mu'^2 \mathbf{1}_{2n'} + h'^2\mu'_\phi\Phi'_{2n'}|^2 + b'|h'^2\mu'|^2 \text{tr } \Phi'_{2n'}\Phi'_{2n'}$$

where $b = \frac{(\ln 4 - 1)}{8\pi^2} \tilde{N}_c$ and $b' = \frac{(\ln 4 - 1)}{8\pi^2} \tilde{N}'_c$ [1, 27]. Differentiating this potential with respect to Φ_{2n} and $\Phi'_{2n'}$ and putting $y\varphi' = 0 = \varphi y$ [3], one obtains

$$\begin{aligned} h\Phi_{2n} &\simeq \frac{\mu_\phi}{b} \mathbf{1}_n \otimes \sigma_3 & \text{or} & & M_{2n} &\simeq \frac{\alpha\Lambda^3}{\tilde{N}_c} \mathbf{1}_n \otimes \sigma_3, \\ h'\Phi'_{2n'} &\simeq \frac{\mu'_\phi}{b'} \mathbf{1}_{n'} \otimes i\sigma_2 & \text{or} & & M'_{2n'} &\simeq \frac{\alpha'\Lambda'^3}{\tilde{N}'_c} \mathbf{1}_{n'} \otimes i\sigma_2 \end{aligned}$$

corresponding to the w coordinates of $2n$ curved flavor D4-branes between the $D6_{-\theta}$ -branes and the NS5'-brane and the w coordinates of $2n'$ curved flavor D4-branes between the $D6_{-\theta'}$ -branes and the NS5'-brane respectively.

3.3 Higher order superpotential for bifundamental

In general, there are also different kinds of meson fields $M_j = Q(XX)^j Q$, $M'_j = Q'(XX)^j Q'$ and $P_r = Q(XX)^r X Q'$ where $j = 0, 1, \dots, 2k' + 1$ and $r = 0, 1, \dots, 2k'$ for higher order term for the bifundamental with general rotation angles of two $(2k' + 1)$ NS5'-branes [14]. The magnetic superpotential contains the interaction between these meson fields with q, q' and Y [14] as well as the higher order term $(YY)^{2k'+1}$ and mass term YY which will vanish for small β, m_X limit. When the N_f $D6_{-\theta}$ -branes and the extra right $2k'$ NS5'-branes do not create the D4-branes and N'_f $D6_{-\theta'}$ -branes and the extra left $2k'$ NS5'-branes also do not create the D4-branes (and their mirrors), the extra meson fields $M_j, j \neq 0$ and P_r do not appear in the magnetic superpotential.

Then the k' -dependent reduced magnetic superpotential with the limit $\beta, m_X \rightarrow 0$ is given by

$$W_{dual} = \left[M_0 q' (YY)^{2k'+1} q' + \frac{\alpha}{2} \text{tr} M_0^2 - m M_0 \right] + \left[M'_0 q (YY)^{2k'+1} q + \frac{\alpha'}{2} \text{tr} M_0'^2 - m' M'_0 \right].$$

Then the analysis for the previous single NS5'-brane case can be performed in this case. One deforms the nonsupersymmetric brane configuration, which is generalized Figure 3 with an orientifold 4-plane (and there are multiple NS5'-branes), by displacing the multiple $D6_{-\theta, -\theta'}$ -branes and the left and right NS5'-branes along v direction [19, 20]. Then the n curved flavor D4-branes attached to them are displaced also as k' different n_j 's connecting between $D6_{-\theta, j}$ -brane and $NS5'_{R, j}$ -brane. Similarly, the n' curved flavor D4-branes attached to them (as well as other D4-branes) are displaced also as k' different n'_j 's connecting between $D6_{-\theta', j}$ -brane and $NS5'_{L, j}$ -brane. When we rescale the submeson fields as $M_j = h\Lambda\Phi_j$ and $M'_j = h'\Lambda'\Phi'_j$, then the Kahler potential for Φ_j and Φ'_j is canonical and the magnetic quarks q_j and q'_j are canonical near the origin of field space [1]. Then the magnetic superpotential can be rewritten

in terms of $\Phi_j, q_j, \Phi'_j, q'_j, Y_j$ and $Y_{j'}$. In order to see the nonsupersymmetric meta-stable ground states, one can split some of the components of $h\Phi_0, q'(YY)^{2k'+1}q', h'\Phi'_0$ and $q(YY)^{2k'+1}q$. The supersymmetric ground state corresponds to

$$\begin{aligned} h\Phi_{2n_j} &= \frac{\mu_j^2}{\mu_\phi} \mathbf{1}_{n_j} \otimes \sigma_3, & \varphi'_{n_j} y_{n_j}^{2k'+1} = 0 = y_{n_j}^{2k'+1} \varphi'_{n_j} & \text{ and} \\ h'\Phi'_{2n'_j} &= \frac{\mu_j'^2}{\mu_\phi'} \mathbf{1}_{n'_j} \otimes i\sigma_2, & y_{n'_j}^{2k'+1} \varphi_{n'_j} = 0 = \varphi_{n'_j} y_{n'_j}^{2k'+1}. \end{aligned}$$

The full one loop potential can be written similarly and the local nonzero stable point arises as

$$h\Phi_{2n_j} \simeq \frac{\mu_\phi}{b_j} \mathbf{1}_{n_j} \otimes \sigma_3 \quad \text{and} \quad h'\Phi'_{2n'_j} \simeq \frac{\mu_\phi'}{b'_j} \mathbf{1}_{n'_j} \otimes i\sigma_2$$

corresponding to the w coordinates of $2n_j$ curved flavor D4-branes between the $D6_{-\theta,j}$ -branes and the $NS5'_{R,j}$ -brane and the w coordinates of $2n'_j$ curved flavor D4-branes between the $D6_{-\theta',j}$ -branes and the $NS5'_{L,j}$ -brane respectively.

4 $SU(N_c) \times SO(N'_c)$ with N_f -fund., $2N'_f$ -vectors and bi-fund.

Let us add an orientifold 6-plane to the brane configuration of section 2 and one extra NS5-brane is needed for the mirror.

4.1 Electric theory

The type IIA supersymmetric electric brane configuration [13, 23] corresponding to $\mathcal{N} = 1$ $SU(N_c) \times SO(N'_c)$ gauge theory with N_f -fundamental flavors Q, \tilde{Q} , $2N'_f$ -vectors Q' and bifundamentals X, \tilde{X} can be described as two NS5-branes, two NS5'-branes, N_c - and N'_c -D4-branes, $2N_f$ - and $2N'_f$ -D6-branes and $O6^+$ -plane. The mass terms for the flavors can be achieved by displacing the D6-branes along $\pm v$ direction leading to their coordinates $v = \pm v_{D6_{-\theta}} (\pm v_{D6_{-\theta'}})$ respectively while the quartic terms for the flavors can be obtained by rotating the D6-branes by an angle $-\theta(-\theta')$ in (w, v) -plane respectively. Then, in the electric gauge theory, the general superpotential is given by

$$\begin{aligned} W_{elec} &= \frac{\alpha}{2} \text{tr}(Q\tilde{Q})^2 - m \text{tr} Q\tilde{Q} + \frac{\alpha'}{2} \text{tr}(Q'Q')^2 - m' \text{tr} Q'Q' \\ &+ \left[\beta_1 \text{tr}(X\tilde{X})^2 + \beta_2 \text{tr} X\tilde{X}\tilde{X}X + \beta_3 (\text{tr} X\tilde{X})^2 + m_X \text{tr} X\tilde{X} \right]. \end{aligned} \quad (4.1)$$

The last four terms of (4.1) are due to the rotation angles ω and ω' of NS5-branes and NS5'-branes where β_i with $i = 1, 2, 3$ depend on ω and ω' and the relative displacement of D4-branes where the mass term for the bifundamental $m_X \equiv \frac{v_{NS5'}}{2\pi\ell_s^2}$ is the distance along the v -direction. We focus on the case $\beta_i (i = 1, 2, 3), m_X \rightarrow 0$.

Let us summarize the $\mathcal{N} = 1$ supersymmetric electric brane configuration for the superpotential (4.1) in type IIA string theory as follows and draw this in Figure 4:

- Two NS5-branes in (012345) directions
- Two NS5'-branes in (012389) directions
- N_f $D6_{\pm\theta}$ -branes in (01237) directions and two other directions in (v, w) -plane
- N'_f $D6_{\pm\theta'}$ -branes in (01237) directions and two other directions in (v, w) -plane
- N_c - and N'_c -color D4-branes in (01236) directions
- $O6^+$ -plane in (0123789) directions

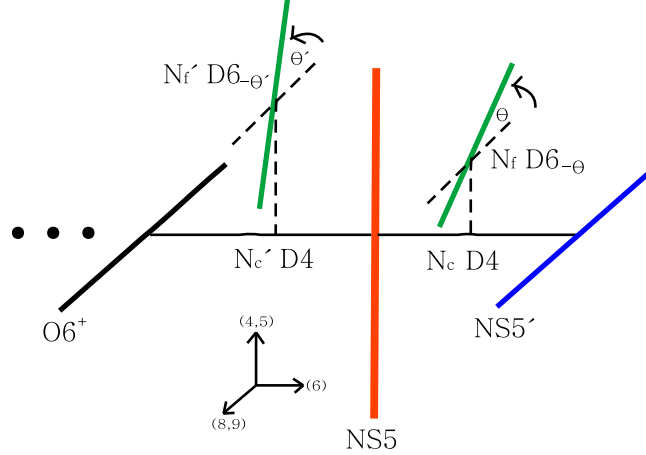


Figure 4: The $\mathcal{N} = 1$ supersymmetric electric brane configuration for the gauge group $SU(N_c) \times SO(N'_c)$ with bifundamentals X, \tilde{X} and fundamentals Q, \tilde{Q} and vector Q' . A rotation of $N_f(N'_f)$ D6-branes in (w, v) -plane corresponds to a quartic term for the fundamentals $Q, \tilde{Q}(Q')$ while a displacement of $N_f(N'_f)$ D6-branes in $\pm v$ direction corresponds to a mass term for the fundamentals $Q, \tilde{Q}(Q')$. The mirrors located at the left hand side of O6-plane and denoted by \dots are preserved under the O6-plane.

4.2 Magnetic theory

The left NS5'-brane starts out with linking number $l_e = -\frac{(N_f)}{2} - \frac{4}{2} + N_c$ and after duality this left NS5'-brane ends up with linking number $l_m = \frac{(N_f)}{2} + \frac{4}{2} + N_f - \tilde{N}_c$. In general, when the left N_f $D6_{-\theta}$ -branes meet the left NS5-brane during the dual process, the new D4-branes are created because they are not parallel. However, we consider only the particular brane motion

where the left N_f $D6_{-\theta}$ -branes meet the left NS5-brane with no angles. In other words, the $D6_{-\theta}$ -branes become $D6_{-\frac{\pi}{2}}$ -branes when they meet with left NS5-brane instantaneously and after that then they come back to the original $D6_{-\theta}$ -branes. Therefore, in this dual process, there is no creation of D4-branes.

Also when the left N_f $D6_{-\theta}$ -branes meet the right NS5'-brane during the dual process, the new D4-branes are created because they are not parallel. However, we consider only the particular brane motion where the left N_f $D6_{-\theta}$ -branes meet the right NS5'-brane with no angles. That is, the $D6_{-\theta}$ -branes become D6-branes when they meet with right NS5'-brane instantaneously and then after that they come back to the original $D6_{-\theta}$ -branes. Similarly, when the left N'_f $D6_{-\theta'}$ -branes meet the right NS5'-brane during the dual process, the new D4-branes are created because they are not parallel. However, we consider only the particular brane motion where the left N'_f $D6_{-\theta'}$ -branes meet the right NS5'-brane with no angles. In other words, the $D6_{-\theta'}$ -branes become D6-branes when they meet with right NS5'-brane instantaneously and then they come back to the original $D6_{-\theta'}$ -branes. Then it turns out that the dual color number \tilde{N}_c is given by $\tilde{N}_c = 2N_f - N_c + 4$.

What about the other dual color number? The left NS5-brane starts out with linking number $l_e = \frac{-N'_f}{2} + N'_c - N_c - \frac{4}{2}$ and after duality this left NS5-brane ends up with linking number $l_m = \frac{N_f^2}{2} + \tilde{N}_c + N'_f - \tilde{N}_c + \tilde{N}'_c + \frac{4}{2}$. Then it turns out that the dual color number \tilde{N}'_c is given by $\tilde{N}'_c = 2N'_f + 2N_f - N'_c + 8$. Finally, one has the following dual color numbers

$$\tilde{N}_c = 2N_f - N_c + 4, \quad \tilde{N}'_c = 2N'_f + 2N_f - N'_c + 8.$$

The low energy theory on the color D4-branes has $SU(\tilde{N}_c) \times SO(\tilde{N}'_c)$ gauge group and N_f -fundamental dual quarks q, \tilde{q} coming from 4-4 strings connecting between the color \tilde{N}_c D4-branes and N_f flavor D4-branes, $2N'_f$ -vectors dual quarks q' coming from 4-4 strings connecting between the color \tilde{N}'_c D4-branes and $2N'_f$ flavor D4-branes as well as Y, \tilde{Y} and various gauge singlets. The $2N_f$ flavors q and \tilde{q} and $2N'_f$ flavors q' are in the representation $(\square, \mathbf{1}), (\bar{\square}, \mathbf{1})$ and $(\mathbf{1}, \square)$ respectively under the gauge group and in the representation $(\bar{\square}, \mathbf{1}, \mathbf{1}), (\mathbf{1}, \bar{\square}, \mathbf{1})$ and $(\mathbf{1}, \mathbf{1}, \bar{\square})$ respectively under the flavor group $SU(N_f)_L \times SU(N_f)_R \times SU(2N'_f)$. In particular, a magnetic meson field

$$M_0 \equiv Q\tilde{Q}$$

is $N_f \times N_f$ matrix and comes from 4-4 strings of N_f flavor D4-branes while a magnetic meson field

$$M'_0 \equiv Q'Q'$$

is $2N'_f \times 2N'_f$ symmetric matrix and comes from 4-4 strings of $2N'_f$ flavor D4-branes. Then the general magnetic superpotential is given by

$$\begin{aligned}
W_{dual} = & \left[\text{tr}(Y\tilde{Y})^2 + \text{tr} Y\tilde{Y}\tilde{Y}Y + (\text{tr} Y\tilde{Y})^2 + \text{tr} Y\tilde{Y} \right. \\
& + M_0 \tilde{q}Y\tilde{Y}Y\tilde{Y}q + M'_0 q'Y\tilde{Y}\tilde{Y}Yq' + M_1 \tilde{q}Y\tilde{Y}q + M'_1 q'Y\tilde{Y}q' + M_2 \tilde{q}q + M'_2 q'q' \\
& + P_0 q\tilde{Y}Y\tilde{Y}q' + P_1 q\tilde{Y}q' + \tilde{P}_0 \tilde{q}Y\tilde{Y}Yq' + \tilde{P}_1 \tilde{q}Yq' + Rq\tilde{Y}\tilde{Y}q + \tilde{R}\tilde{q}Y\tilde{Y}\tilde{q} \left. \right] \\
& + \frac{\alpha}{2} \text{tr} M_0^2 - mM_0 + \frac{\alpha'}{2} \text{tr} M_0'^2 - m'M_0'. \tag{4.2}
\end{aligned}$$

Here the mesons are $M_1 \equiv Q\tilde{X}X\tilde{Q}$, $M'_1 \equiv Q'X\tilde{X}Q'$, $M_2 \equiv Q\tilde{X}X\tilde{X}X\tilde{Q}$, $M'_2 \equiv Q'X\tilde{X}\tilde{X}XQ'$ and $P_0 \equiv Q\tilde{X}Q'$, $\tilde{P}_0 \equiv \tilde{Q}XQ'$, $P_1 \equiv Q\tilde{X}X\tilde{X}Q'$, $\tilde{P}_1 \equiv \tilde{Q}X\tilde{X}XQ'$, $R \equiv Q\tilde{X}\tilde{X}Q$, $\tilde{R} \equiv \tilde{Q}X\tilde{X}\tilde{Q}$. Note that the mesons of the $SU(N_c)$ group couple to the dual quarks of $SU(\tilde{N}_c)$ and the mesons of the $SO(N'_c)$ group couple to the dual quarks of $SO(\tilde{N}'_c)$, compared with the unitary case in section 2. The first three lines of (4.2) is present in [13].

As in section 2, one can dualize each gauge group independently of the other. One labels each NS5-branes from left to right A, B, C and D [13]. For example, one dualizes the first gauge group factor by moving the B NS5-brane to the left of the A NS5'-brane while the C NS5-brane to the right of the D NS5'-brane like as in [23]. Then the intermediate dual gauge group is given by $SU(\tilde{n}_c = N_f + N'_c - N_c) \times SO(N'_c)$ with corresponding superpotential for the dual matters. Now we interchange the A- and D- NS5'-branes and $D6_{-\theta, -\theta'}$ -branes each other and obtain the intermediate dual gauge group $SU(\tilde{n}_c) \times SO(\tilde{n}'_c = 2N'_f + 4N_f + N'_c - 2N_c + 4)$ with dual matters. Next, we move the D NS5'-brane and $D6_{-\theta, -\theta'}$ -branes to the left of the B NS5-brane while the A NS5'-brane and $D6_{-\theta, -\theta'}$ -branes to the right of the C NS5-brane and obtain the intermediate dual gauge group $SU(\tilde{N}_c = 4N_f + 2N'_f - N_c + 4) \times SO(\tilde{n}'_c)$ with corresponding superpotential. Then, the B- and C- NS5-brane are interchanged each other through O6-plane and we obtain the dual gauge group $SU(\tilde{N}_c) \times SO(\tilde{N}'_c = 4N_f + 4N'_f - N'_c + 8)$ with superpotential (4.2) ⁶.

As we explained before, our particular brane motion during the dual process does not produce any D4-branes when the left N_f $D6_{-\theta}$ -branes meet the left NS5-brane. This implies that there is no M_2 term in the above superpotential (4.2). We consider only the particular brane motion where the left N_f $D6_{-\theta}$ -branes meet the right NS5'-brane with no angles. This implies that there is no M_1, R or \tilde{R} term in the above superpotential (4.2). By construction, there are no additional D4-branes connecting the left NS5-brane and N'_f $D6_{-\theta'}$ -branes after duality. This leads to the fact that there is no M'_1 or M'_2 dependence in (4.2) ⁷. Furthermore,

⁶ It does not matter the order of dualization. We obtain the same dual gauge theory if we start with $SO(N'_c)$ dualization, then $SU(N_c)$ dualization, $SO(\tilde{n}'_c)$ dualization and end up with $SU(\tilde{n}_c)$ dualization.

⁷In [13], they introduced the $4N'_f$ full D4-branes without changing the linking number in order to satisfy

we do not see any P_1 - or \tilde{P}_1 -dependent terms in the superpotential (4.2) when the left N_f $D6_{-\theta}$ -branes, the right N'_f $D6_{-\theta'}$ -branes and the left NS5'-brane during the dual process meet each other with no angles after first $SU(N_c)$ dualization ⁸. When the left N_f $D6_{-\theta}$ -branes, the right N'_f $D6_{-\theta'}$ -branes and the left NS5'-brane during the dual process meet each other with no angles after first $SO(N'_c)$ dualization, we do not see any P_0 - or \tilde{P}_0 -dependent terms in the superpotential (4.2) ⁹. Similarly, we do not see any R - or \tilde{R} -dependent terms in the superpotential (4.2) ¹⁰. Then the reduced magnetic superpotential with the limit $\beta_1, \beta_2, \beta_3, m_X \rightarrow 0$ is given by

$$W_{dual} = \left[M_0 \tilde{q} Y \tilde{Y} Y \tilde{Y} q + \frac{\alpha}{2} \text{tr} M_0^2 - m M_0 \right] + \left[M'_0 q' Y \tilde{Y} \tilde{Y} Y q' + \frac{\alpha'}{2} \text{tr} M_0'^2 - m' M'_0 \right]. \quad (4.3)$$

For the supersymmetric vacua, one can compute the F-term equations for this superpotential (4.3) and the expectation values for $M_0, M'_0, \tilde{q} Y \tilde{Y} Y \tilde{Y} q$ and $q' Y \tilde{Y} \tilde{Y} Y q'$ are obtained. The F-term equations for $M_0, q, \tilde{q}, M'_0, q', Y$ and \tilde{Y} are

$$\begin{aligned} \tilde{q} Y \tilde{Y} Y \tilde{Y} q - m + \alpha M_0 &= 0, & (M_0 \tilde{q} Y \tilde{Y}) Y \tilde{Y} &= 0, & Y \tilde{Y} (Y \tilde{Y} q M_0) &= 0, \\ q' Y \tilde{Y} \tilde{Y} Y q' - m' + \alpha' M'_0 &= 0, & (M'_0 q' Y \tilde{Y}) \tilde{Y} Y + Y \tilde{Y} (\tilde{Y} Y q' M'_0) &= 0, \\ Y [\tilde{Y} \tilde{Y} q M_0] \tilde{q} + \tilde{Y} (Y \tilde{Y} q M_0) \tilde{q} + q' (M'_0 q' Y \tilde{Y}) \tilde{Y} + \tilde{Y} (\tilde{Y} Y q' M'_0) q' &= 0, \\ q (M_0 \tilde{q} Y \tilde{Y}) Y + q [M_0 \tilde{q} Y Y] \tilde{Y} + 2Y (\tilde{Y} Y q' M'_0) q' &= 0. \end{aligned} \quad (4.4)$$

the correct dual color numbers in the gauge theory side analysis. This has led to the fact that there are $3N'_f$ D4-branes connecting the left NS5-brane and N'_f $D6_{-\theta'}$ -branes (and their mirrors) after duality. In other words, the extra $2N'_f$ D4-branes were needed for the meson fields M'_1 and M'_2 . In our construction, we do not need these extra the $4N'_f$ full D4-branes because we do not want to have these unwanted meson fields M'_1 and M'_2 .

⁸These mesons P_1 and \tilde{P}_1 originate from $SU(N_c)$ chiral mesons $Q\tilde{X}$ and $\tilde{Q}X$ [23] when one dualizes the first gauge group factor first. That is, the strings stretching between the N_f “flavor” D4-branes and N'_c color D4-branes give rise to these $2N_f$ $SO(N'_c)$ vectors. The superpotential in [23] contains the cubic term between dual bifundamental, these meson fields and dual quarks. After three additional dual procedures, $SO(N'_c), SU(\tilde{n}_c), SO(\tilde{n}'_c)$, these cubic terms arise as P_1 and \tilde{P}_1 -term in (4.2) where there exist extra quarks q' -dependence while P_1 and \tilde{P}_1 have extra $X\tilde{X}Q'$ and $\tilde{X}XQ'$ fields respectively, due to the further $SO(N'_c)$ and $SO(\tilde{n}'_c)$ -dualizations.

⁹These mesons P_0 and \tilde{P}_0 originate from $SO(N'_c)$ chiral mesons $\tilde{X}Q'$ and XQ' when one dualizes the second gauge group factor. That is, the strings stretching between the N'_f “flavor” D4-branes and N_c color D4-branes give rise to these $2N'_f$ $SU(N_c)$ fundamentals. The superpotential contains the cubic term between dual bifundamental, these meson fields and dual quarks. After three additional dual procedures, $SU(N_c), SO(\tilde{n}'_c), SU(\tilde{n}_c)$, these cubic terms arise as P_0 and \tilde{P}_0 -term in (4.2) where there exist extra factors $q\tilde{Y}Y$ and $\tilde{q}Y\tilde{Y}$ respectively while P_0 and \tilde{P}_0 have extra Q and \tilde{Q} fields respectively, due to the further $SU(N_c)$ and $SU(\tilde{n}_c)$ -dualizations.

¹⁰These mesons R and \tilde{R} originate from $SU(N_c)$ chiral mesons $\tilde{X}Q$ and $X\tilde{Q}$ when one dualizes the first gauge group factor first. That is, the strings stretching between the N_f “flavor” D4-branes and N'_c color D4-branes give rise to these $2N_f$ $SO(N'_c)$ vectors as before. The superpotential contains the cubic term between dual bifundamental, these meson fields and dual quarks. After three additional dual procedures, $SO(N'_c), SU(\tilde{n}_c), SO(\tilde{n}'_c)$, these cubic terms arise as R and \tilde{R} -term in (4.2) where there exist extra factors $q\tilde{Y}$ and $\tilde{q}Y$ while R and \tilde{R} have extra $Q\tilde{X}$ and $\tilde{Q}X$ fields, due to the further $SU(\tilde{n}_c)$ -dualization.

The sixth and seventh equations of (4.4) are satisfied if the the second, third, fifth equations of (4.4) hold: $M_0 \tilde{q} Y \tilde{Y} = Y \tilde{Y} q M_0 = M'_0 q' Y \tilde{Y} = \tilde{Y} Y q' M'_0 = 0$ and $\tilde{Y} \tilde{Y} q M_0 = M_0 \tilde{q} Y Y = 0$. Any vacuum expectation values for the fields should obey this condition. We present the magnetic brane configuration in Figure 5.

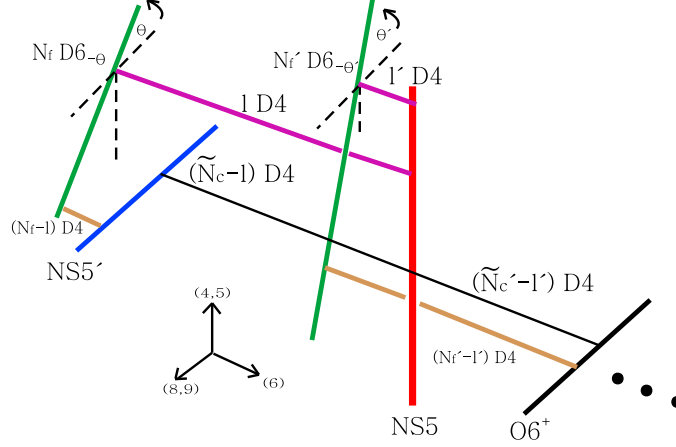


Figure 5: The $\mathcal{N} = 1$ supersymmetric magnetic brane configuration corresponding to Figure 4 with a splitting and a reconnection between D4-branes when the gravitational potential of the NS5-brane is ignored. The N_f flavor D4-branes connecting between $D6_{-\theta}$ -branes and NS5'-brane are splitting into $(N_f - l)$ - and l - D4-branes. The N'_f flavor D4-branes connecting between $D6_{-\theta'}$ -branes and NS5'-brane are splitting into $(N'_f - l')$ - and l' - D4-branes. The mirrors denoted by \cdots are assumed in this figure.

The theory has many nonsupersymmetric meta-stable ground states and when we rescale the meson field as $M_0 = h\Lambda\Phi_0$ and $M'_0 = h'\Lambda'\Phi'_0$, then the Kahler potential for Φ_0 and Φ'_0 is canonical and the magnetic quarks are canonical near the origin of field space [1]. Then the magnetic superpotential can be rewritten as

$$W_{mag} = \left[h\Phi_0 \tilde{q} Y \tilde{Y} Y \tilde{Y} q + \frac{h^2 \mu_\phi}{2} \text{tr } \Phi_0^2 - h\mu^2 \text{tr } \Phi_0 \right] + \left[h'\Phi'_0 q' Y \tilde{Y} \tilde{Y} Y q' + \frac{h'^2 \mu'_\phi}{2} \text{tr } \Phi_0'^2 - h'\mu'^2 \text{tr } \Phi'_0 \right]$$

where $\mu^2 = m\Lambda, \mu'^2 = m'\Lambda'$ and $\mu_\phi = \alpha\Lambda^2, \mu'_\phi = \alpha'\Lambda'^2$.

Now one splits the $(N_f - l) \times (N_f - l)$ block at the lower right corner of $h\Phi_0$ and $\tilde{q} Y \tilde{Y} Y \tilde{Y} q$ into blocks of size n and $(N_f - l - n)$ and one decomposes the $2(N'_f - l') \times 2(N'_f - l')$ block at the lower right corner of $h'\Phi'_0$ and $q' Y \tilde{Y} \tilde{Y} Y q'$ into blocks of size $2n'$ and $2(N'_f - l' - n')$ as

follows [3]:

$$\begin{aligned}
h\Phi_0 &= \begin{pmatrix} 0_l & 0 & 0 \\ 0 & h\Phi_n & 0 \\ 0 & 0 & \frac{\mu^2}{\mu_\phi} \mathbf{1}_{N_f-l-n} \end{pmatrix}, & h'\Phi'_0 &= \begin{pmatrix} 0_{2l'} & 0 & 0 \\ 0 & h'\Phi'_{2n'} & 0 \\ 0 & 0 & \frac{\mu'^2}{\mu'_\phi} \mathbf{1}_{N'_f-l'-n'} \otimes \sigma_3 \end{pmatrix}, \\
\tilde{q}Y\tilde{Y}Y\tilde{Y}q &= \begin{pmatrix} \mu^2 \mathbf{1}_l & 0 & 0 \\ 0 & \tilde{\varphi}y\tilde{y}y\tilde{y}\varphi & 0 \\ 0 & 0 & 0_{N_f-l-n} \end{pmatrix}, \\
q'Y\tilde{Y}Y\tilde{Y}q' &= \begin{pmatrix} \mu'^2 \mathbf{1}_{2l'} & 0 & 0 \\ 0 & \varphi'y\tilde{y}y\tilde{y}\varphi' & 0 \\ 0 & 0 & 0_{2(N'_f-l'-n')} \end{pmatrix}.
\end{aligned}$$

Here φ and $\tilde{\varphi}$ are $n \times (\tilde{N}_c - l)$ dimensional matrices and correspond to n flavors of fundamentals of the gauge group $SU(\tilde{N}_c - l)$ which is unbroken and φ' is $2n' \times (\tilde{N}'_c - l')$ dimensional matrices and correspond to $2n'$ flavors of fundamentals of the gauge group $SO(\tilde{N}'_c - l')$ which is unbroken [3, 4]. The φ and $\tilde{\varphi}$ correspond to fundamental strings connecting the n flavor D4-branes and $(\tilde{N}_c - l)$ color D4-branes and φ' corresponds to fundamental strings connecting the $2n'$ flavor D4-branes and $(\tilde{N}'_c - l')$ color D4-branes. The Φ_n and $\tilde{\varphi}y\tilde{y}y\tilde{y}\varphi$ are $n \times n$ matrices while $\Phi'_{2n'}$ and $\varphi'y\tilde{y}y\tilde{y}\varphi'$ are $2n' \times 2n'$ matrices. The supersymmetric ground state corresponds to

$$h\Phi_n = \frac{\mu^2}{\mu_\phi} \mathbf{1}_n, \quad \tilde{\varphi}y\tilde{y} = 0 = y\tilde{y}\varphi \quad \text{and} \quad h'\Phi'_{2n'} = \frac{\mu'^2}{\mu'_\phi} \mathbf{1}_{2n'} \otimes \sigma_3, \quad \varphi'y\tilde{y} = 0 = \tilde{y}y\varphi'.$$

Now the full one loop potential from the superpotential and vacuum expectation values for the fields takes the form

$$\begin{aligned}
V &= |h\Phi_n \tilde{\varphi}y\tilde{y}|^2 + |hy\tilde{y}\varphi\Phi_n|^2 + |h\tilde{\varphi}y\tilde{y}y\tilde{y}\varphi - h\mu^2 \mathbf{1}_n + h^2 \mu_\phi \Phi_n|^2 + b|h^2 \mu|^2 \text{tr} \Phi_n^\dagger \Phi_n \\
&+ |h'\Phi'_{2n'} \varphi'y\tilde{y}|^2 + |\tilde{y}y\varphi'\Phi'_{2n'}|^2 + |h'\varphi'y\tilde{y}y\tilde{y}\varphi' - h'\mu'^2 \mathbf{1}_{2n'} + h'^2 \mu'_\phi \Phi'_{2n'}|^2 \\
&+ b'|h'^2 \mu'|^2 \text{tr} \Phi'_{2n'} \Phi'_{2n'} + |h\tilde{y}\tilde{y}\varphi\Phi_n|^2 + |h\Phi_n \tilde{\varphi}y\tilde{y}|^2
\end{aligned}$$

where $b = \frac{(\ln 4 - 1)}{8\pi^2} \tilde{N}_c$ and $b' = \frac{(\ln 4 - 1)}{8\pi^2} \tilde{N}'_c$ [1, 27]. Differentiating this potential with respect to Φ_n^\dagger and $\Phi'_{2n'}$ and putting $\tilde{y}\varphi = 0 = \tilde{\varphi}y$ and $\varphi'y = 0 = y\varphi'$, one obtains

$$\begin{aligned}
h\Phi_n &\simeq \frac{\mu_\phi}{b} \mathbf{1}_n \quad \text{or} \quad M_n \simeq \frac{\alpha \Lambda^3}{\tilde{N}_c} \mathbf{1}_n, \\
h'\Phi'_{2n'} &\simeq \frac{\mu'_\phi}{b'} \mathbf{1}_{2n'} \otimes \sigma_3 \quad \text{or} \quad M'_{2n'} \simeq \frac{\alpha' \Lambda'^3}{\tilde{N}'_c} \mathbf{1}_{2n'} \otimes \sigma_3
\end{aligned}$$

corresponding to the w coordinates of n curved flavor D4-branes between the $D6_{-\theta}$ -branes and the NS5'-brane and the w coordinates of $2n'$ curved flavor D4-branes between the $D6_{-\theta'}$ -branes and the NS5'-brane respectively.

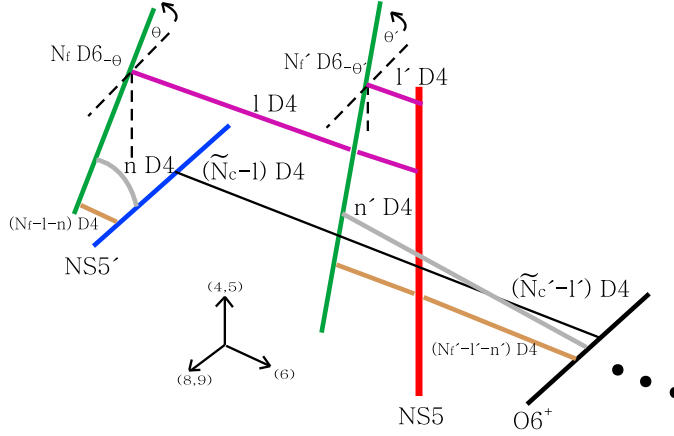


Figure 6: The nonsupersymmetric meta-stable magnetic brane configuration corresponding to Figure 4 with a misalignment between D4-branes when the gravitational potential of the NS5-brane is considered. The $(N_f - l)$ flavor D4-branes in Figure 5 connecting between $D6_{-\theta}$ -branes and NS5'-brane are splitting into $(N_f - l - n)$ - and n - D4-branes. The $(N_f' - l')$ flavor D4-branes in Figure 5 connecting between $D6_{-\theta'}$ -branes and NS5'-brane are splitting into $(N_f' - l' - n')$ - and n' - D4-branes. We do not present the mirrors denoted by \cdots for simplicity.

4.3 $SU(N_c) \times Sp(N_c')$ with N_f - and N_f' -fund. and bifund.

In this case, the corresponding electric brane configuration can be read off from Figure 4 by changing the $O6^+$ -plane into $O6^-$ -plane and the NS5-brane(NS5'-brane) into the NS5'-brane(NS5-brane). One analyzes the magnetic theory by using the method of the previous case and obtains the corresponding reduced magnetic superpotential where M'_0 is an antisymmetric matrix. Also the nonsupersymmetric brane configuration can be obtained from the Figure 6 by changing the role of NS5-brane and NS5'-brane.

5 Conclusions and outlook

In this paper, we presented the meta-stable brane configurations for the product gauge groups by dualizing the whole gauge groups. In the context of supersymmetric brane configurations, there are also different kinds of dual theories we did not study in this work, from the gauge theory side or IIA string theory side. For example, the theory with adjoint fields, the theory of generalization of [28], the gauge theory with triple product gauge groups which does not have corresponding field theory analysis or higher product gauge groups [29, 30], the theory with product gauge groups when the orientifold 6-plane is located at the NS5-brane or the NS5'-brane [31], or the theory with no D6-branes [32, 33, 34, 35]. It would be interesting to discover whether the meta-stable brane configurations exist.

Acknowledgments

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References

- [1] K. Intriligator, N. Seiberg and D. Shih, “Dynamical SUSY breaking in meta-stable vacua,” JHEP **0604**, 021 (2006) [arXiv:hep-th/0602239].
- [2] K. Intriligator and N. Seiberg, “Lectures on Supersymmetry Breaking,” Class. Quant. Grav. **24**, S741 (2007) [arXiv:hep-ph/0702069].
- [3] A. Giveon and D. Kutasov, “Stable and Metastable Vacua in SQCD,” Nucl. Phys. B **796**, 25 (2008) [arXiv:0710.0894 [hep-th]].
- [4] A. Giveon and D. Kutasov, “Stable and Metastable Vacua in Brane Constructions of SQCD,” JHEP **0802**, 038 (2008) [arXiv:0710.1833 [hep-th]].
- [5] A. Giveon and D. Kutasov, “Gauge symmetry and supersymmetry breaking from intersecting branes,” Nucl. Phys. B **778**, 129 (2007) [arXiv:hep-th/0703135].
- [6] A. Giveon and D. Kutasov, “Brane dynamics and gauge theory,” Rev. Mod. Phys. **71**, 983 (1999) [arXiv:hep-th/9802067].
- [7] H. Ooguri and Y. Ookouchi, “Meta-stable supersymmetry breaking vacua on intersecting branes,” Phys. Lett. B **641**, 323 (2006) [arXiv:hep-th/0607183].
- [8] S. Franco, I. Garcia-Etxebarria and A. M. Uranga, “Non-supersymmetric meta-stable vacua from brane configurations,” JHEP **0701**, 085 (2007) [arXiv:hep-th/0607218].
- [9] I. Bena, E. Gorbatov, S. Hellerman, N. Seiberg and D. Shih, “A note on (meta)stable brane configurations in MQCD,” JHEP **0611**, 088 (2006) [arXiv:hep-th/0608157].
- [10] J. H. Brodie and A. Hanany, “Type IIA superstrings, chiral symmetry, and $N = 1$ 4D gauge theory dualities,” Nucl. Phys. B **506**, 157 (1997) [arXiv:hep-th/9704043].
- [11] R. Tatar, “Dualities in 4D theories with product gauge groups from brane configurations,” Phys. Lett. B **419**, 99 (1998) [arXiv:hep-th/9704198].
- [12] C. Ahn, K. Oh and R. Tatar, “Branes, geometry and $N = 1$ duality with product gauge groups of SO and Sp,” J. Geom. Phys. **31**, 301 (1999) [arXiv:hep-th/9707027].

- [13] E. Lopez and B. Ormsby, “Duality for $SU \times SO$ and $SU \times Sp$ via branes,” JHEP **9811**, 020 (1998) [arXiv:hep-th/9808125].
- [14] K. A. Intriligator, R. G. Leigh and M. J. Strassler, “New examples of duality in chiral and nonchiral supersymmetric gauge theories,” Nucl. Phys. B **456**, 567 (1995) [arXiv:hep-th/9506148].
- [15] E. Barnes, K. Intriligator, B. Wecht and J. Wright, “ $N = 1$ RG flows, product groups, and a-maximization,” Nucl. Phys. B **716**, 33 (2005) [arXiv:hep-th/0502049].
- [16] C. Ahn, “Other Meta-Stable Brane Configuration by Adding an Orientifold 6-Plane to Giveon-Kutasov,” arXiv:0712.0032 [hep-th].
- [17] C. Ahn, “More on Meta-Stable Brane Configuration by Quartic Superpotential for Fundamentals,” arXiv:0801.0257 [hep-th], to appear in PLB.
- [18] C. Ahn, “Meta-Stable Brane Configurations by Quartic Superpotential for Bifundamentals,” arXiv:0801.0619 [hep-th].
- [19] C. Ahn, “Meta-Stable Brane Configurations by Higher Order Polynomial Superpotential,” arXiv:0802.0044 [hep-th].
- [20] C. Ahn, “Meta-Stable Brane Configurations, Multiple NS5-Branes, and Rotated D6-Branes,” arXiv:0803.0836 [hep-th].
- [21] I. Brunner, A. Hanany, A. Karch and D. Lust, “Brane dynamics and chiral non-chiral transitions,” Nucl. Phys. B **528**, 197 (1998) [arXiv:hep-th/9801017].
- [22] C. Ahn and R. Tatar, “Geometry, D-branes and $N = 1$ duality in four dimensions with product gauge groups,” Phys. Lett. B **413**, 293 (1997) [arXiv:hep-th/9705106].
- [23] C. Ahn, “Meta-stable brane configuration of product gauge groups,” arXiv:0704.0121 [hep-th], to appear in CQG.
- [24] C. Ahn, “Meta-stable brane configuration with orientifold 6 plane,” JHEP **0705**, 053 (2007) [arXiv:hep-th/0701145].
- [25] C. Ahn, “More on Meta-Stable Brane Configuration,” Class. Quant. Grav. **24**, 3603 (2007) [arXiv:hep-th/0702038].
- [26] C. Ahn, “Meta-Stable Brane Configuration and Gauged Flavor Symmetry,” Mod. Phys. Lett. A **22**, 2329 (2007) [arXiv:hep-th/0703015].

- [27] C. Ahn, “M-theory lift of meta-stable brane configuration in symplectic and orthogonal gauge groups,” *Phys. Lett. B* **647**, 493 (2007) [arXiv:hep-th/0610025].
- [28] C. Ahn, “Brane configurations for nonsupersymmetric meta-stable vacua in SQCD with adjoint matter,” *Class. Quant. Grav.* **24**, 1359 (2007) [arXiv:hep-th/0608160].
- [29] C. Ahn, “Meta-Stable Brane Configurations of Triple Product Gauge Groups,” arXiv:0708.4255 [hep-th].
- [30] C. Ahn, “Meta-Stable Brane Configurations of Multiple Product Gauge Groups with Orientifold 6 Plane,” arXiv:0710.0180 [hep-th].
- [31] C. Ahn, “Meta-Stable Brane Configurations with Five NS5-Branes,” arXiv:0705.0056 [hep-th].
- [32] C. Ahn, “Meta-Stable Brane Configurations by Adding an Orientifold-Plane to Giveon-Kutasov,” *JHEP* **0708**, 021 (2007) [arXiv:0706.0042 [hep-th]].
- [33] C. Ahn, “More Meta-Stable Brane Configurations without D6-Branes,” *Nucl. Phys. B* **790**, 281 (2008) [arXiv:0707.0092 [hep-th]].
- [34] C. Ahn, “Meta-Stable Brane Configurations with Seven NS5-Branes,” arXiv:0708.0439 [hep-th], to appear in CQG.
- [35] C. Ahn, “Meta-Stable Brane Configurations with Multiple NS5-Branes,” arXiv:0711.0082 [hep-th].